The Effect of Different Local Search Algorithms on the Performance of Multi-Objective Optimizers (Supplementary Material)

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***Abstract*—Several contemporary multi-objective surrogate-based algorithms use some kind of local search operator. The search technique used in this operator can largely affect the performance of the multi-objective optimizer as a whole, however, little attention is often paid to the selection of this technique. In this paper, we compare three different local search techniques and evaluate their effect on the performance of two different surrogate based multi-objective optimizers. The algorithms are evaluated using the well known ZDT and WFG benchmark suites and recommendations are made based on the results.**

I. Introduction

This document contains supplementary materials for the paper submitted to the 2014 IEEE Congress on Evolutionary Algorithms. These are more detailed results, which would not fit in the original paper due to space limitations. See that paper (with the same name as this one) for details on the methods and experiments.

In all the tables and graphs presented in this document the names of the algorithms consist of two parts – the first part expresses the type of algorithm (ASM for ASM-MOMA, and svmHO and pmHO for HO-MOMA with SVM based model and perfect model respectively). The second part, separated by a dash from the first part, denotes the type of local search ("Grad" for gradient search, "Rand" for random search, and "EA" for simple evolutionary algorithm).

The graphs show the median and first and third quartile (as error bars) computed from 15 independent runs.

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Table I

COMPARISON OF ASM-MOMA WITH DIFFERENT LOCAL SEARCH ALGORITHMS ON THE ZDT AND WFG BENCHMARK. ΔH VALUES AFTER GIVEN NUMBER OF FUNCTION EVALUATIONS, MEDIAN OVER 15 RUNS. SUPERSCRIPT INDICATES THAT THE GIVEN VERSION IS SIGNIFICANTLY (MANN-WHITNEY U-TEST = WILCOXON RANK-SUM TEST, $p \leq 0.001$) BETTER THAN THE VERSION INDICATED BY THE SUPERSCRIPT ("R" FOR RANDOM SEARCH, "G" FOR GRADIENT SEARCH, "E" FOR EVOLUTIONARY ALGORITHM).

Test	Algorithm	Function evaluations				
		1000	5000	10000	20000	30000
ZDT1	ASM-Rand	6.74583	0.02200	0.00176	0.00039	0.00020
	ASM-Grad	8.00935	0.01726	0.00174	0.00044	0.00019
	ASM-EA	7.07308	0.02718	0.00108	0.00035	0.00019
ZDT2	ASM-Rand	22.52573	9.92765	6.13713	0.00203	0.00056
	ASM-Grad	24.44211	9.73422	3.10672	0.00464	0.00090
	ASM-EA	23.48211	9.47326	5.11252	0.00178	0.00053
ZDT3	ASM-Rand	9.08498	0.03307	0.00161	0.00016	0.00007
	ASM-Grad	9.15679	0.02025	0.00148	0.00018	0.00008
	ASM-EA	10.46014	0.02136	0.00132	0.00018	0.00007
ZDT4	ASM-Rand	120.66213	69.67198	18.50852	1.39742	0.05118
	ASM-Grad	120.66213	65.58476	21.41127	2.01258	0.07671
	ASM-EA	120.66213	82.11839	22.68206	2.33871	0.05292
ZDT6	ASM-Rand	8.82739	0.12201	0.03369	0.00718	0.00684
	ASM-Grad	9.11109	0.10493	0.03500	0.00690	0.00689
	ASM-EA	9.69432	0.17297	0.03380	0.00757	0.00682
WFG1	ASM-Rand	10.29437	9.35879	8.28187	6.72132	5.54470
	ASM-Grad	10.29195	9.40478	8.28530	6.62224	5.47968
	ASM-EA	10.26842	9.40535	8.32684	6.46705	5.48610
WFG2	ASM-Rand	3.42027	1.62309	1.19278	0.96623	0.91434
	ASM-Grad	3.40511	1.61464	1.21604	0.96927	0.90981
	ASM-EA	3.50105	1.71938	1.22775	0.95537	0.90108
WFG3	ASM-Rand	2.96779	0.80875	0.35228	0.14097	0.09216
	ASM-Grad	2.93827	0.81629	0.32780	0.12634	0.07368
	ASM-EA	2.95505	0.80206	0.33574	0.12417	0.07808
WFG4	ASM-Rand	2.21710	0.75265	0.40807	0.16872	0.08840
	ASM-Grad	2.28448	0.80360	0.42118	0.20820	0.11022
	ASM-EA	2.22754	0.75667	0.41803	0.19643	0.09726
WFG5	ASM-Rand	1.60891	0.65145	0.60481	0.56535	0.55649
	ASM-Grad	1.50626	0.65959	0.60121	0.57658	0.55655
	ASM-EA	1.64112	0.64835	0.60175	0.56927	0.55921
WFG6	ASM-Rand	3.38836	1.05493	0.61882	0.43203	0.40250
	ASM-Grad	3.46641	1.11366	0.67574	0.49709	0.40660
	ASM-EA	3.40516	1.10676	0.68689	0.48485	0.43844
WFG7	ASM-Rand	2.60904	0.73562	0.33083	0.11950	0.06001
	ASM-Grad	2.63039	0.71479	0.23635	0.09655	0.06034
	ASM-EA	2.55869	0.68693	0.25156	0.09466	0.05262
WFG8	ASM-Rand	3.40405	1.86113	1.50274	1.23278	1.14557
	ASM-Grad	3.56764	1.93106	1.55065	1.27621	1.15008
	ASM-EA	3.50801	1.92694	1.54785	1.25835	1.15414
WFG9	ASM-Rand	2.96173	1.06514	1.00366	1.00013	0.99160
	ASM-Grad	3.05103	1.03714	1.00321	0.98968	0.98860
	ASM-EA	2.75802	1.06372	1.00509	0.99603	0.99064

Figure 1. Convergence graphs of ASM-MOMA with different local search algoritms.

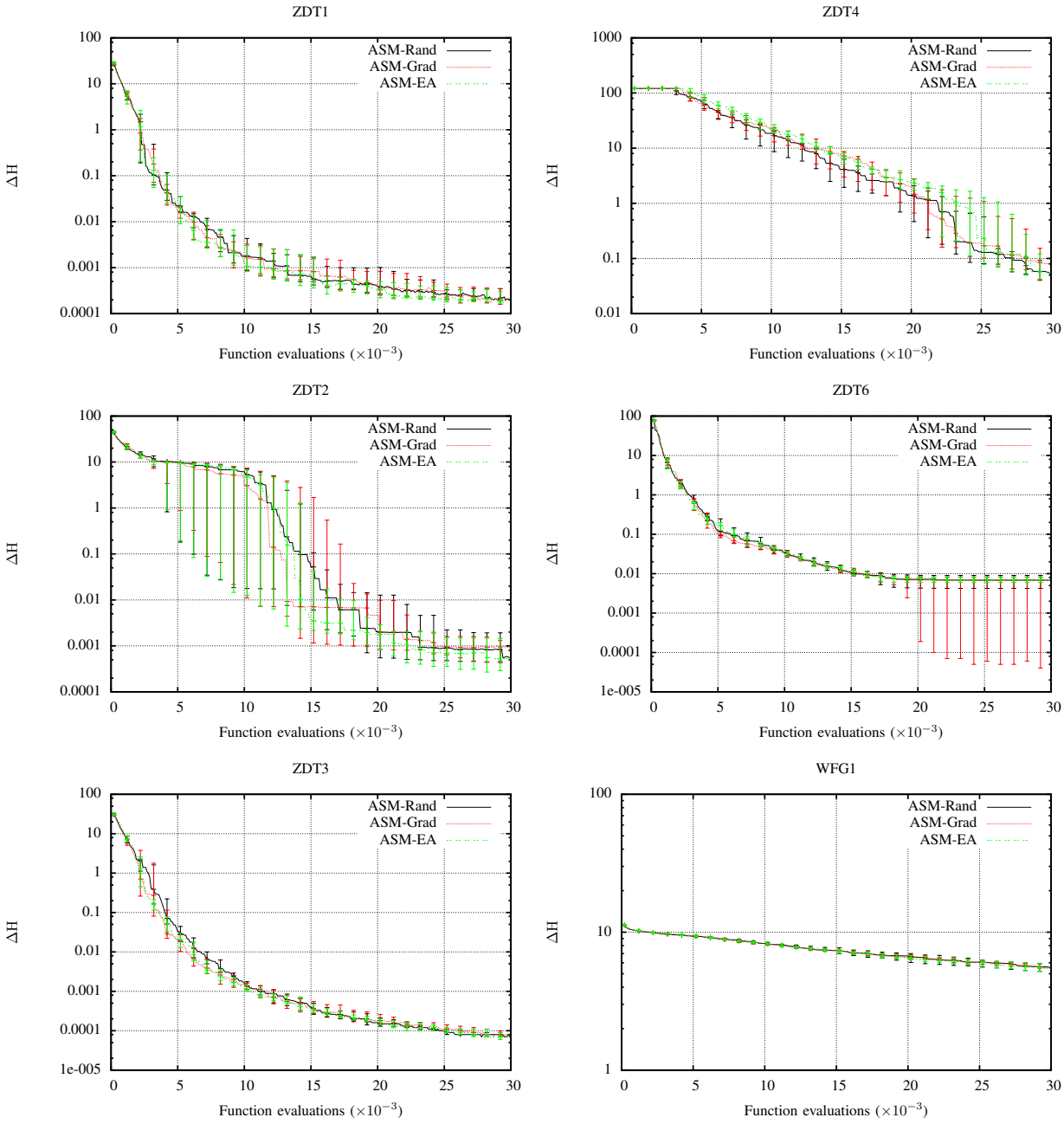

Figure 2. Convergence graphs of ASM-MOMA with different local search algoritms.

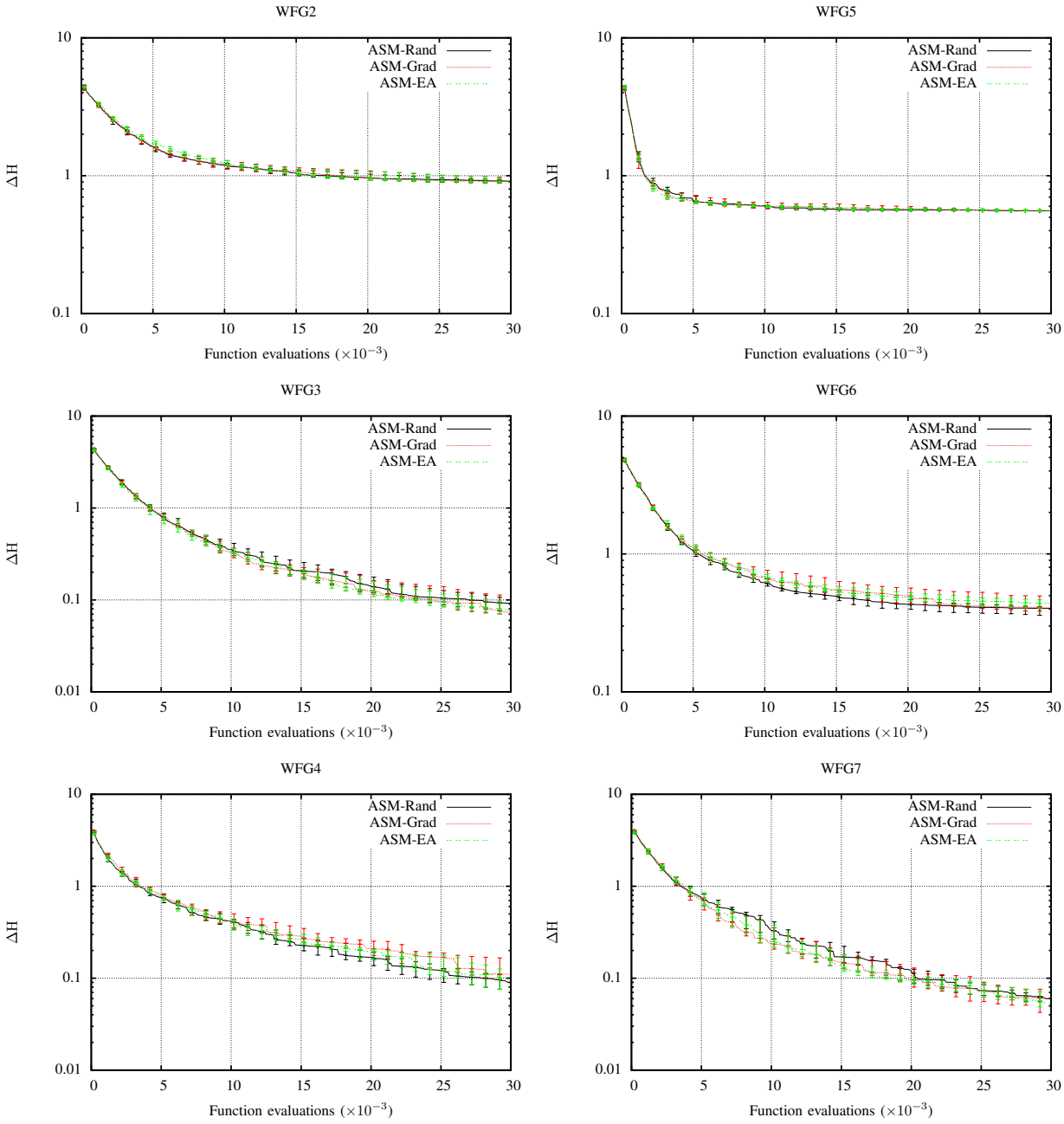

Figure 3. Convergence graphs of ASM-MOMA with different local search algoritms.

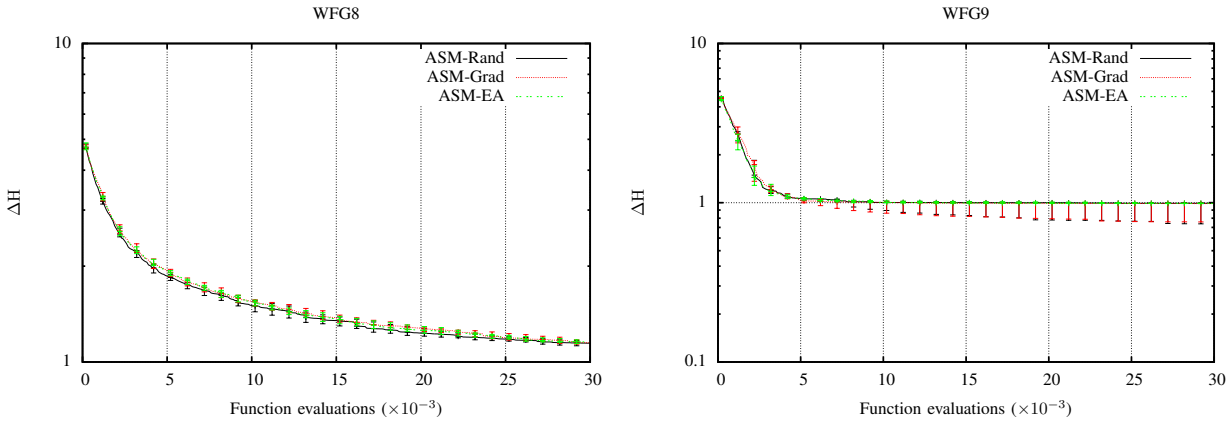

Table II

COMPARISON OF DIFFERNET LOCAL SEARCH ALGORITHMS IN HO-MOMA WITH SVM-BASED MODEL ON THE ZDT AND WFG BECHMARK. .ΔH VALUES AFTER GIVEN NUMBER OF FUNCTION EVALUATIONS, MEDIAN OVER 15 RUNS. SUPERSCRIPT INDICATES THAT THE GIVEN VERSION IS SIGNIFICANTLY (MANN-WHITNEY U-TEST = WILCOXON RANK-SUM TEST, $p \leq 0.001$) BETTER THAN THE VERSION INDICATED BY THE SUPERSCRIPT ("R" FOR RANDOM SEARCH, "G" FOR GRADIENT SEARCH, "E" FOR EVOLUTIONARY ALGORITHM).

Test	Algorithm	Function evaluations				
		1000	5000	10000	20000	30000
ZDT1	svmHO-Rand	9.10025	0.11731	0.02289	0.00402	0.00148
	svmHO-Grad	1.64973^{R}	0.00018^{RE}	0.00018^{RE}	0.00017^{RE}	0.00017^{R}
	svmHO-EA	0.57926^{RG}	0.00572^{R}	0.00104^{R}	0.00028^{R}	0.00019^{R}
ZDT2	svmHO-Rand	25.55882	0.45108	0.06089	0.01062	0.00570
	svmHO-Grad	9.42744^{R}	0.00015^{RE}	0.00016^{RE}	0.00014^{RE}	0.00015^{R}
	svmHO-EA	10.51132^{R}	0.00724^{R}	0.00140^{R}	0.00029^{R}	0.00015^{R}
ZDT3	svmHO-Rand	13.75323	0.28247	0.03960	0.00601	0.00194
	svmHO-Grad	10.17445^{R}	0.45080	0.00328	0.00011^{RE}	0.00007^{RE}
	svmHO-EA	2.02444^{RG}	0.03557^{RG}	0.00615^{R}	0.00109^{R}	0.00037^{R}
ZDT4	svmHO-Rand	120.66213	33.30602	7.18286	0.23636	0.04188
	svmHO-Grad	120.66213	22.91896^{E}	3.13822^{E}	0.16253^{E}	0.00587^{RE}
	svmHO-EA	120.66213	51.53511	11.54795	1.70093	0.07724
ZDT6	svmHO-Rand	24.88189	0.00019^{E}	0.00008^{E}	0.00005^{GE}	0.00004^{GE}
	svmHO-Grad	0.13156^{RE}	0.00011^{RE}	0.00010^{E}	0.00009^{E}	0.00008^{E}
	svmHO-EA	7.44843	0.03781	0.01335	0.01058	0.01036
WFG1	svmHO-Rand	6.85460^{GE}	6.59004^{E}	6.35491^{E}	5.29995^{E}	4.32314^{E}
	svmHO-Grad	10.40650	6.41803^{RE}	5.63014^{RE}	4.74714^{RE}	3.90252^{RE}
	svmHO-EA	9.99846	9.38267	8.68669	7.19572	6.35357
WFG2	svmHO-Rand	3.31184	1.63450	1.20832	1.00195^{G}	0.92478^{G}
	svmHO-Grad	3.13734	1.60591	1.34968	1.15917	1.05533
	svmHO-EA	3.10093^{R}	1.03026^{RG}	0.50633^{RG}	0.22948^{RG}	0.13635^{RG}
WFG3	svmHO-Rand	2.74103	0.95892^{G}	0.46007^{G}	0.20180^{G}	0.12455^{G}
	svmHO-Grad	2.96213	1.28632	0.68432	0.35623	0.17518
	svmHO-EA	2.64890^{G}	0.84459^{G}	0.40321^{G}	0.16968^{G}	0.09658^{G}
WFG4	svmHO-Rand	1.88982^{GE}	0.66963^{GE}	0.38224^{GE}	0.19357	0.12158
	svmHO-Grad	2.41692	1.00482	0.56712	0.27177	0.15701
	svmHO-EA	2.17738	0.92429	0.49185	0.23242	0.14353
WFG5	svmHO-Rand	2.52484^{G}	0.84915^{G}	0.66176^{G}	0.58836	0.57098
	svmHO-Grad	2.89528	1.23513	0.76407	0.60703	0.58272
	svmHO-EA	2.03412^{RG}	0.84300^{G}	0.67074^{G}	0.59822	0.57580
WFG6	svmHO-Rand	3.21382	1.21653	0.70206	0.44661^{G}	0.37006^{G}
	svmHO-Grad	3.23972	1.28697	0.84150	0.63907	0.54493
	svmHO-EA	3.46120	1.26807	0.79755	0.49595	0.42179^{G}
WFG7	svmHO-Rand	2.40409	0.66110^{G}	0.25870^{G}	0.09748^{G}	0.05958^{G}
	svmHO-Grad	2.59338	1.14038	0.60903	0.33862	0.14848
	svmHO-EA	2.33807^{G}	0.68880^{G}	0.31844^{G}	0.11779^{G}	0.07809^{G}
WFG8	svmHO-Rand	3.19427^{G}	1.87655^{G}	1.51033^{G}	1.28009^{G}	1.18389^{G}
	svmHO-Grad	3.39607	2.21232	1.83015	1.44584	1.28410
	svmHO-EA	3.06935^{G}	1.84973^{G}	1.54074^{G}	1.30112^{G}	1.19581^{G}
WFG9	svmHO-Rand	2.16099^{G}	1.08479	1.02081	0.99852	0.99572
	svmHO-Grad	2.75849	1.10105	0.99366	0.98561	0.98477
	svmHO-EA	2.55647	1.10684	1.01438	1.00072	0.99067

Figure 4. Convergence graphs of HO-MOMA with SVM-based model and with different local search algoritms.

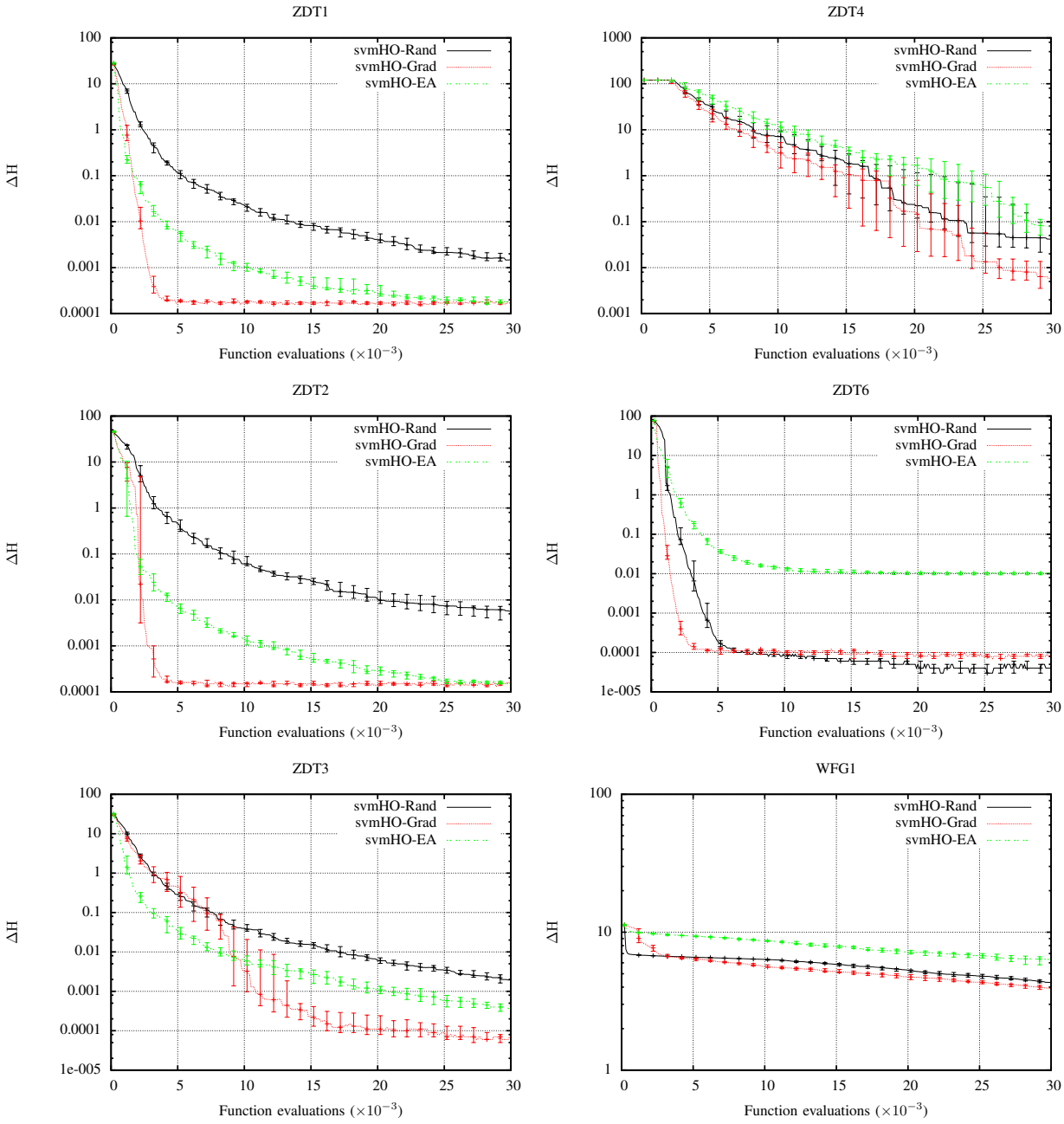

Figure 5. Convergence graphs of HO-MOMA with SVM-based model and with different local search algoritms.

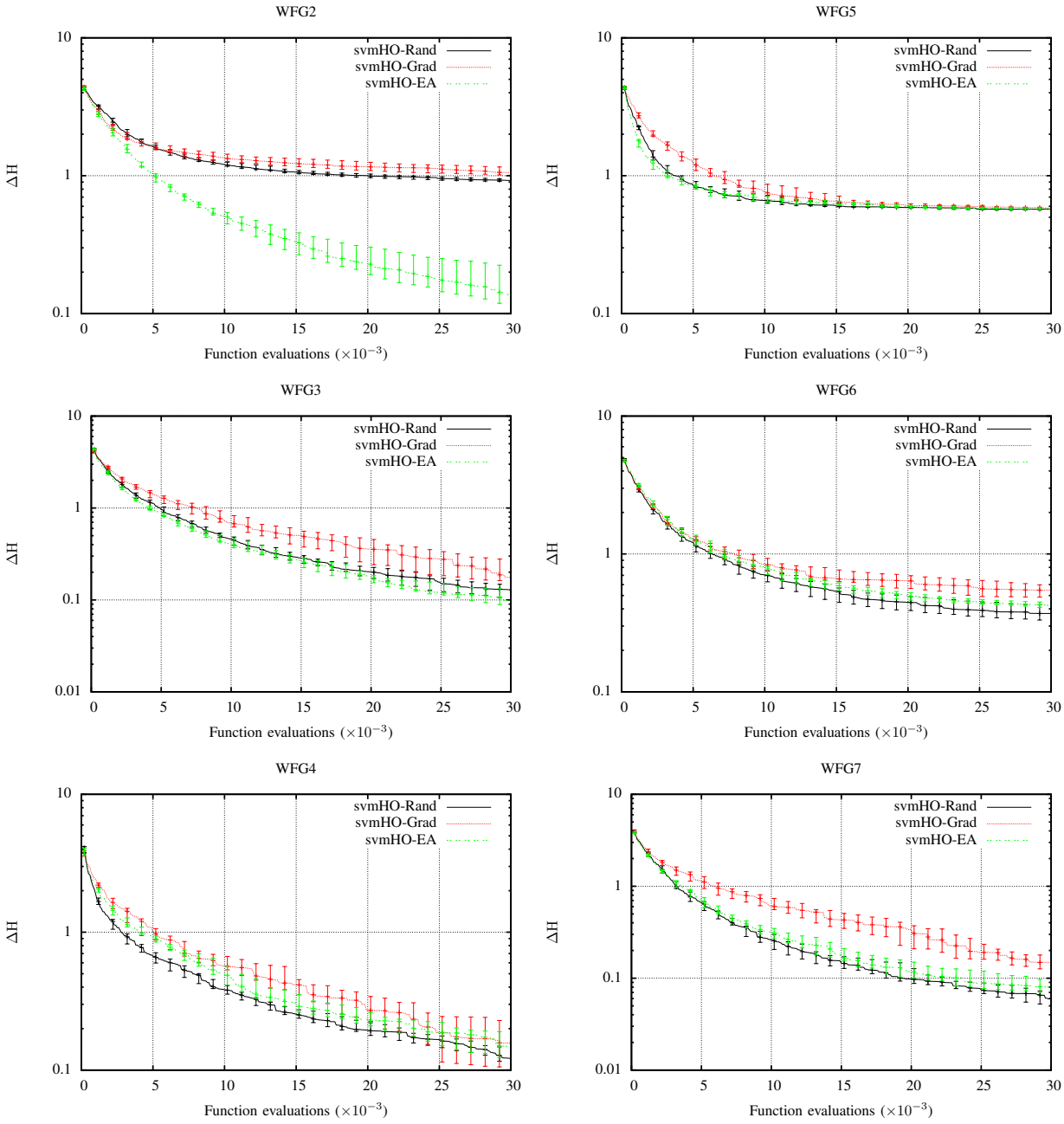

Figure 6. Convergence graphs of HO-MOMA with SVM-based model and with different local search algoritms.

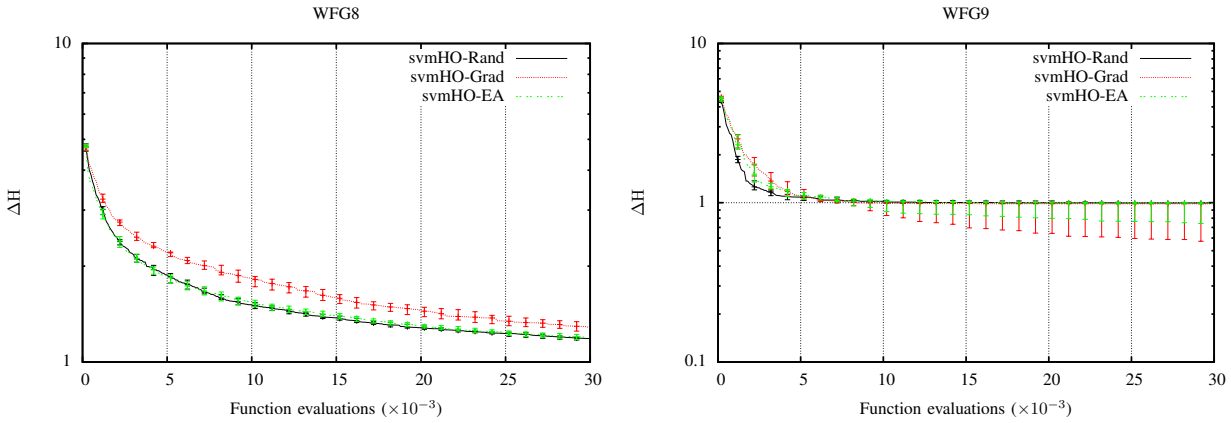

Table III

COMPARISON DIFFERENT LOCAL SEARCH ALGORITHMS IN HO-MOMA WITH PERFECT MODEL ON THE ZDT AND WFG BECHMARK. ΔH VALUES AFTER GIVEN NUMBER OF FUNCTION EVALUATIONS, MEDIAN OVER 15 RUNS. SUPERSCRIPT INDICATES THAT THE GIVEN VERSION IS SIGNIFICANTLY (MANN-WHITNEY U-TEST = WILCOXON RANK-SUM TEST, $p \leq 0.001$) BETTER THAN THE VERSION INDICATED BY THE SUPERSCRIPT ("R" FOR RANDOM SEARCH, "G" FOR GRADIENT SEARCH, "E" FOR EVOLUTIONARY ALGORITHM).

Test	Algorithm	Function evaluations				
		1000	5000	10000	20000	30000
ZDT1	pmHO-Rand	8.63967	0.13151	0.02609	0.00408	0.00153
	pmHO-Grad	1.78385^{R}	0.00018^{RE}	0.00017^{RE}	0.00017^{RE}	0.00017^{RE}
	pmHO-EA	0.09436^{RG}	0.01797^{R}	0.00462^{R}	0.00081^{R}	0.00053^{R}
ZDT2	pmHO-Rand	24.75238	0.33931	0.06298	0.01271	0.00391
	pmHO-Grad	16.06045^{R}	0.00015^{RE}	0.00014^{RE}	0.00015^{RE}	0.00014^{RE}
	pmHO-EA	1.06423^{RG}	0.01878^{R}	0.00994^{R}	0.00183^{R}	0.00102^{R}
ZDT3	pmHO-Rand	11.24405	0.19768	0.03575	0.00545	0.00200
	pmHO-Grad	5.13948^{R}	0.00491^{RE}	0.00031^{RE}	0.00007^{RE}	0.00006^{RE}
	pmHO-EA	0.29634^{RG}	0.03891^{R}	0.01566	0.00169^{R}	0.00055^{R}
ZDT4	pmHO-Rand	120.66213	19.72420	5.61226	0.40642	0.07729
	pmHO-Grad	120.66213	24.40925	2.61510^{R}	0.00016^{RE}	0.00018^{RE}
	pmHO-EA	13.47859^{RG}	0.03132^{RG}	0.02242^{RG}	0.00914^{R}	0.00184^{R}
ZDT6	pmHO-Rand	34.38397	0.00014^{GE}	0.00008^{E}	0.00006^{E}	0.00003^{GE}
	pmHO-Grad	1.02972^{RE}	0.00027^{E}	0.00008^{E}	0.00007^{E}	0.00005^{E}
	pmHO-EA	3.72776	0.02494	0.01628	0.01195	0.01049
WFG1	pmHO-Rand	7.22537^{G}	6.72758^{G}	6.36691^{G}	5.27560	4.17718
	pmHO-Grad	10.64724	7.31778	6.63269	5.08875	3.92622
	pmHO-EA	6.59073^{RG}	4.23818^{RG}	2.97806^{RG}	1.96651^{RG}	1.45348^{RG}
WFG2	pmHO-Rand	2.49340^{G}	1.26165^{G}	1.05236^{G}	0.93883^{G}	0.90271^{G}
	pmHO-Grad	3.13038	1.63788	1.38273	1.15015	1.06007
	pmHO-EA	1.41015^{RG}	0.96110^{RG}	0.94233^{RG}	0.93209^{G}	0.91815
WFG3	pmHO-Rand	2.27141^{G}	0.70458^{G}	0.33234^{G}	0.13894^{G}	0.08432^{G}
	pmHO-Grad	2.71982	1.23620	0.70909	0.32649	0.20114
	pmHO-EA	0.56785^{RG}	0.20534^{RG}	0.15409^{RG}	0.09718^{G}	0.07724^{G}
WFG4	pmHO-Rand	1.52774^{G}	0.55894^{G}	0.31709	0.15784	0.09328
	pmHO-Grad	2.07514	0.71910	0.33674	0.12095	0.08450
	pmHO-EA	0.25139^{RG}	0.11117^{RG}	0.08005^{RG}	0.04226^{RG}	0.03046^{RG}
WFG5	pmHO-Rand	1.66632^{G}	0.73579^{G}	0.64157^{G}	0.57529^{G}	0.56842^{G}
	pmHO-Grad	2.85454	1.18262	0.78423	0.62249	0.58731
	pmHO-EA	0.66326^{RG}	0.59316^{RG}	0.56889^{RG}	0.55867^{RG}	0.55357^{G}
WFG6	pmHO-Rand	2.50255^{G}	1.16867^{G}	0.71633	0.43447^{G}	0.34901^{G}
	pmHO-Grad	3.32392	1.37829	0.87487	0.59882	0.52919
	pmHO-EA	1.00229^{RG}	0.60294^{RG}	0.48601^{RG}	0.43875^{G}	0.40147
WFG7	pmHO-Rand	2.04137^{G}	0.54245^{G}	0.22210^{G}	0.09282^{G}	0.05844
	pmHO-Grad	2.59977	1.12601	0.64338	0.19603	0.07349
	pmHO-EA	0.21192^{RG}	0.08093^{RG}	0.04666^{RG}	0.02627^{RG}	0.01710^{RG}
WFG8	pmHO-Rand	2.83154^{G}	1.72888^{G}	1.43235^{G}	1.22015^{G}	1.14244^{G}
	pmHO-Grad	3.43977	2.14726	1.78390	1.44588	1.28381
	pmHO-EA	1.39388^{RG}	0.97480^{RG}	0.92328^{RG}	0.87957^{RG}	0.86493^{RG}
WFG9	pmHO-Rand	1.30803^{G}	1.04181	1.01142	1.00430	0.99309
	pmHO-Grad	1.58466	1.03090	0.99943	0.99185	0.98771
	pmHO-EA	0.66651^{RG}	0.45105^{RG}	0.41040^{RG}	0.36325^{RG}	0.34769^{RG}

Figure 7. Convergence graphs of HO-MOMA with perfect model and with different local search algoritms.

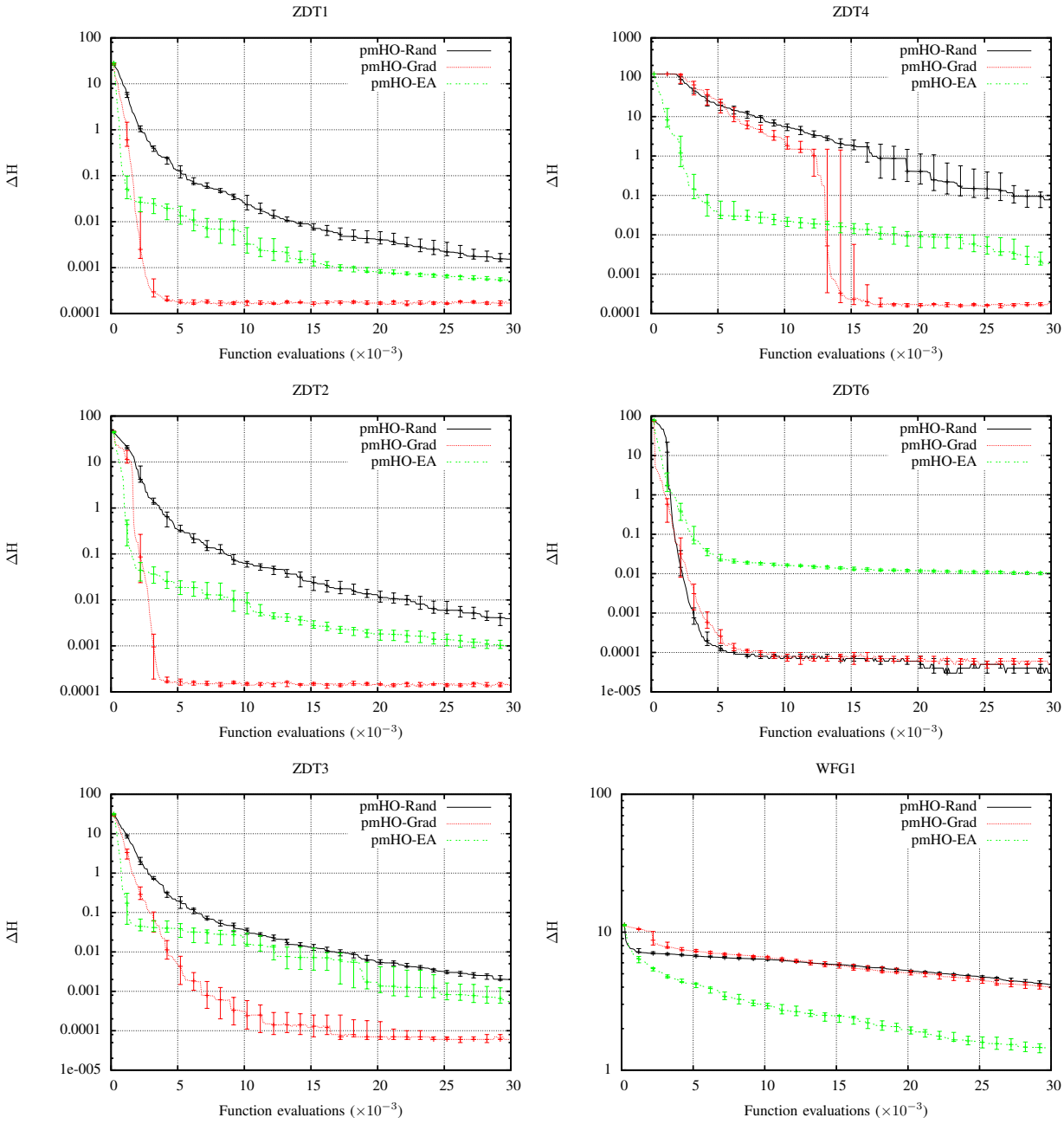

Figure 8. Convergence graphs of HO-MOMA with perfect model and with different local search algoritms.

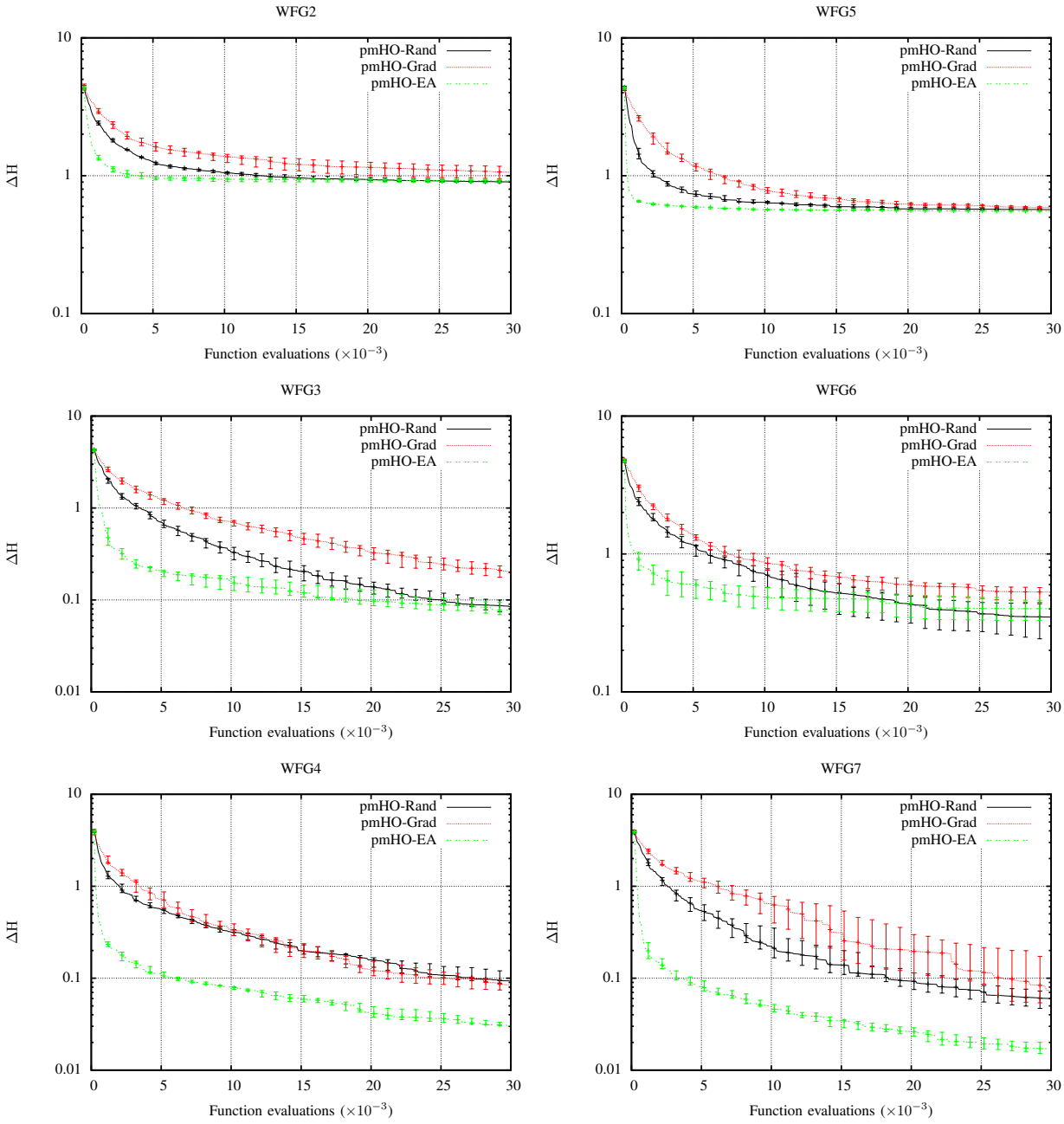

Figure 9. Convergence graphs of HO-MOMA with perfect model and with different local search algoritms.